ENTANGLEMENT INDUCED RADIATION PROCESSES WHICH ARE FIRST ORDER IN WEAK FIELD

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Abstract

Nonlinear processes of light scattering on a two-level system near resonance are considered. The problem is reduced to the emission and absorption of an entangled system, formed by a strong resonant field and a two-level system, having a non-factorizing wave function. Various regimes of emission and absorption of the entangled system are determined by the parameter $\alpha = |2dE/\hbar\Delta|$, where E is the amplitude of the strong resonant field, d is the electric dipole moment of the transition 1-2 of the unperturbed atom,

Δ is the detuning off resonance, $\Delta = E_2 - E_1 - \hbar\omega$, with $E_{1,2}$ being the energies of, respectively, lower and upper atomic levels, and the carrier frequency of the resonant field. Two limiting cases of the switching on the interaction between the atom and field are considered, those of the adiabatic and the sudden switching on; these two cases allow simple analytical solutions and lead to essentially differing physical results, either for the probabilities of the processes of the first order with respect to the weak field, or for their coherent properties. Note that at $\alpha << 1$ the conventional perturbation technique is applicable. This means that for weak fields we can use perturbation method because the corresponding $\alpha' << 1$. The nonlinear resonance fluorescence and the amplification of weak radiation field by the entangled system at optical frequencies are considered. The coherent properties of the emission of entangled system are studied which is important for the problems of propagation in gaseous media, decoherency and quantum communication.

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1.Introduction

Investigations of the optical radiation scattering by single atoms or in macroscopic gaseous media have almost centennial history. These investigations became their further development recently in conjunction with atom cooling and appearance of new, highly effective, lasers. Of special interest is the use of these processes in the problems of creation of one-atom lasers (see, e.g., [1] and references therein), and for quantum computations (see, e.g., [2] and references therein). In the present work we consider processes of the first order, with respect to the weak field, on a coupled system "atom+strong resonant field" which will be termed below as "entangled system" (ES). A special attention will be paid to coherent properties of the emission of ES and to the ways of switching on the interaction between the two-level system and the strong resonant field. The adiabatic and sudden switching on for which the problem has simple solutions are the models for real situations. The parameter determining these two limiting cases is the ratio of the resonance detuning Δ to the temporal characteristics of the pump (e.g., the temporal characteristic, τ, of the pulse leading edge). At $|\Delta\tau| >> 1$ the adiabatic switching is realized while at $|\Delta\tau| << 1$ the switching is sudden. In the first case the atomic electrons are perturbed slightly what leads to the well-known Kramers-Heisenberg formula for the dispersion and Raman-effect [3-6]; in the second case the atomic electrons oscillate, by stimulated emission and absorption of photons of the external resonant field, between the atomic levels (see, e.g., [7] where is a mistake corrected in [8]). In the following calculations we do not introduce phenomenologically the damping, Γ , of the upper level. The reasons for this are following. First, the time of spontaneous electric-dipole transitions is of the order of 10^{-8}s, i.e., at shorter interaction times there is no reason to introduce these transitions; second, it is assumed that $|\Delta| > \Gamma$; third, to calculate the spontaneous processes in an ES we have to use an other technique than that used by V. Weisskopf, [3-6]; finally, the frequency of population oscillations in a strong resonant field which is of order $|dE/\hbar|$ and plays the dominant role in the calculations below should be greater than the natural linewidth Γ . To the best of our knowledge the use of ES in calculations of nonlinear resonance fluorescence and of absorption of a weak external radiation has been suggested in Ref.[9] where the adiabatic switching was studied (when comparing the present results

with those of [9] it should be taken into account that the notations for the roots of the characteristic equation are different). For clarifying the questions of light amplification in scattering by an atomic system (or, in other words, of interference of radiation of ES formed by atoms in gaseous media or in ion traps) the study of the coherence properties of individual components of the scattered radiation is needed. These problems are closely related to those of information transfer and recording by means of atomic systems. In this conjunction we employ here the semiclassical theory of radiation where the phase and amplitude of strong resonant field may be treated simultaneously. To study the wave pictures of radiation processes it can be done [see, for example, [3, 6, 8]. Using, however, the correspondence principle makes it possible to interpret the obtained physical results quantumelectrodynamically as well. Naturally, the method of correspondence [3-8], giving the results equivalent to quantumelectrodynamical calculations, is used for emission and absorption of the photons of a weak radiation. The generalization of the obtained results for the non-classical description of the strong resonant field, when the particles properties of light are important, requires a special investigation.

For adiabatic switching on the interaction with a two-level atom the wave functions of an ES will be denoted below by $\Phi_{1,2}$ while for sudden switching on the interaction by $\Phi'_{1,2}$. As each of these function systems is a complete orthonormilized system describing ES, the formulas connecting these two systems have the following form (see, e.g. [8]).

$$\Phi'_1 = C_1\Phi_1 + C_2^*\Phi_2, \Phi'_2 = -C_2\Phi_1 + C_1\Phi_2 \tag{1}$$

where

$$C_1 = \frac{1}{\sqrt{2}}\left(1 + \frac{\Delta}{\Omega}\right)^{1/2}, C_2 = -\frac{\lambda_1}{V}C_1. \tag{2}$$

The quantities entering C_1 and C_2 are equal to

$$\Delta = E_{21} - \omega, \Omega = \sqrt{\Delta^2 + 4\left|V\right|^2}, \lambda_1 = \frac{\Delta - \Omega}{2}, V = -\frac{Ed_{21}}{\hbar}, \tag{3}$$

with E being the amplitude of the field acting on a two-level atom near resonance, i.e. under the condition

$$|\Delta| << E_{21}, \tag{4}$$

where $E_{21} = E_2 - E_1$ is the difference of the upper and lower levels energies measured in $sec^{-1}(see, Fig.1)$

$$E(r,t) = Ee^{ikr - i\omega t} + c.c. \tag{5}$$

The electric dipole moment of the unperturbed two-level atom is denoted by d_{21}

$$d_{21} = d_{12}^* = \int \psi_2^* e r \psi_1 dV = e^{i(\varphi_1 - \varphi_2)} d, \tag{6}$$

where

$$\psi_{1,2} = u_{1,2} e^{-iE_{1,2}t} \tag{7}$$

are the wave functions of a free atom in the lower and the upper states. Note that for our convenience in formula (6) we separated random phases of the atom as the wave function in quantum mechanics is determined to within a random phase. These phases are different for each single atom and denoted by $\varphi_{1,2}^i$. When calculating the scattering light by an ensemble of atoms it is necessary to average over the random phases. With the results being independent of random phases, such processes are coherent; otherwise, in the emission processes of ES the radiation intensities from different ES will be simply added, i.e. such processes will be noncoherent. With atom being acted upon by a resonance radiation and random phase being different at different moments, the same arguments can be related to one atom. In accordance with the quasiclassical radiation theory the probability of spontaneous emission of the photon with the frequency ω' , which is equal to the difference of the energies of two discrete levels, the momentum k' and the polarization e' into the solid angle dO' in the transition $2 \to 1$ willbe determined by the expression [3-6]

$$dW_{sp} = \frac{\omega'^3}{2\pi\hbar c^3} \left| e'^* d^- \right|^2 dO', \tag{8}$$

where d^- is the negative frequency part of the dipole moment. For a system of atoms, with coordinates r_i expression (8) reads [5]

$$dW_{sp} = \frac{\omega'^3}{2\pi\hbar c^3} \left| \sum_i (e'^* d_i^-) e^{i(k-k')r_i} \right|^2 dO'. \tag{9}$$

Besides spontaneous emission there exist stimulated emission and absorption the probability of which are determined by the same formula (8) multiplied by n', with n' being

the total numberof photons with the energy ω', the momentum k' and the polarization e'. The quantity n' is expressed in terms of the spectral-angular density $I(k',e')$ of stimulating radiation by the following formula

$$n'(k',e') = \frac{8\pi^3 c^2}{\hbar\omega'^3} I(k',e'), \tag{10}$$

where the radiation intensity is equal to $J = \int I(k',e')d\omega' dO'$. In the case of stimulated processes the solid angle should be treated as the solid angle in direction distribution of the stimulated radiation. In this case the probability of emission and absorption will be determined by the following simple expressions, respectively [3-6]

$$dW' = dW_{sp}(n'+1) \tag{11}$$

$$dW' = dW_{sp}n'. \tag{12}$$

With ES being considered instead of a free atom, all the changes to be introduced in formula (8) are reduced to replacing the negative frequency part of the dipole moment of the free atom by the negative frequency part of the ES dipole moment.

2. Coherence of "entangled systems" radiation and comparison of electric dipole moments of "entangled systems" at various regimes of switching

The expressions for D_{ik} in the case of adiabatic switching are given in [9] (see, also [8]) and have the following form

$$D_{11} = -D_{22} = -\frac{V}{\Omega} d_{12} e^{-i\omega t} + c.c. \tag{13}$$

$$D_{12} = D_{21}^* = -\frac{1}{2}\left(1 - \frac{\Delta}{\Omega}\right) d_{21} e^{-i(\omega-\Omega)t - 2i\varphi_0} + \frac{1}{2}\left(1 + \frac{\Delta}{\Omega}\right) d_{12} e^{-i(\omega+\Omega)t}, \tag{14}$$

with $\varphi_0 = \varphi + \varphi_1^i - \varphi_2^i$ being the phase of the interaction

$$V = |V|\, e^{i\varphi_0 - i\pi}, \tag{15}$$

where φ is a constant or a slowly varying phase of the field E; $\varphi_{1,2}^i$ are the above-denoted random phases of the atoms. As it is seen from (13), the latter cancels the random

phase d_{12} in the expression (13) and remains in (14), i.e. the processes described by the dipole moment D_{11} will be coherent ones, while radiative processes induced by the dipole moment D_{12} will be noncoherent. Besides, there is a phase of the radiation in expression (15). This phase, naturally, should not be changed during the interaction. A known example of that type is the "forward" stars light scattering in atmosphere which is coherent and the noncoherent "side" stars light scattering determining the blue color of the sky. The photons reaching the Earth should be coherent or identical, i.e. to be in a phase volume of $\Delta x \Delta y \Delta z \Delta p_x \Delta p_y \Delta p_z \approx (2\pi\hbar)^3$. Having applied this relation to a star on the whole, it is easy to be convinced of the fact that due to giant distances to the Earth the light radiated by stars and detected on the Earth is coherent (see, e.g., [10]).

The dipole moments of ES for that case of sudden switching D'_{ik} are obtained from the relation (1) and the expressions (13), (14).

a) Consider these relations in the limit of $\alpha < 1$. Omitting simple calculations, we have

$$D'_{11} = -D'_{22} = sign\Delta D_{11} - \left(\frac{\alpha}{2} e^{i\varphi_0} D_{21} + c.c.\right) \tag{16}$$

$$D'_{12} = \left(1 - \frac{\alpha^2}{4}\right) D_{12} - e^{-2i\varphi_0} \frac{\alpha^2}{4} D_{21} + \alpha e^{-i\varphi_0} D_{11} at \Delta > 0 \tag{17}$$

$$D'_{12} = \frac{\alpha^2}{4} D_{12} - e^{-2i\varphi_0} \left(1 - \frac{\alpha^2}{4}\right) D_{21} + \alpha e^{-i\varphi_0} D_{11} at \Delta < 0 \tag{18}$$

In accordance with the expression (14) at $\alpha < 1$ we have

$$D_{11} = -D_{22} = \frac{\alpha}{2}\left(1 - \frac{\alpha^2}{2}\right) e^{i\varphi_0 - i\omega t} d_{12} + c.c. \tag{19}$$

$$D_{12} = -\frac{\alpha^2}{4} d_{21} e^{i(\omega-\Omega)t - 2i\varphi_0} + \left(1 - \frac{\alpha^2}{4}\right) d_{12} e^{-i(\omega+\Omega)t} at \Delta > 0 \tag{20}$$

$$D_{12} = -\left(1 - \frac{\alpha^2}{4}\right) d_{21} e^{i(\omega-\Omega)t - 2i\varphi_0} + \frac{\alpha^2}{4} d_{12} e^{-i(\omega+\Omega)t} at \Delta < 0 \tag{21}$$

b)In the case of $\alpha > 1$ the same relations take the following form:

$$D'_{11} = -D'_{22} = \frac{sign\Delta}{\alpha} D_{11} - \frac{1}{2}\left(e^{i\varphi_0} D_{12} + c.c.\right) \tag{22}$$

$$D'_{12} = \frac{1}{2}\left(1 + \frac{sign\Delta}{\alpha}\right) D_{12} - \frac{1}{2} e^{-2i\varphi_0} \left(1 - \frac{sign\Delta}{\alpha}\right) D_{21} + D_{11} e^{-i\varphi_0} \tag{23}$$

D_{11} and D_{12} entering (22) and (23) at $\alpha > 1$ are equal to:

$$D_{11} = -\frac{1}{2}\left(1 - \frac{1}{2\alpha^2}\right)\left(e^{-i\varphi_0 + i\omega t} d_{21} + c.c.\right) \tag{24}$$

$$D_{12} = \frac{1}{2}\left(1 + \frac{sign\Delta}{\alpha}\right) d_{12} e^{-i(\omega+\Omega)t} - \frac{1}{2}\left(1 - \frac{sign\Delta}{\alpha}\right) d_{21} e^{-i(\omega-\Omega)t - 2i\varphi_0} \tag{25}$$

Expressions (16), (19), (22), (24) will determine coherent processes while the expressions (17), (18), (20), (21), (23), (25) noncoherentones. It appears at once from the above that during sudden switching on the interaction there is a new qualitative phenomenon - noncoherent Rayleigh scattering caused by the presence of the D_{11} term initiating Rayleigh scattering in the expressions (17), (18) and (23). On the analogy of that, the term of D_{12} in (16) and (22) causes the appearance of coherent radiation at the $\omega \pm \Omega$ side frequencies, which is always noncoherent during adiabatic switching. During the interaction between the resonant radiation (depending on the way of switching on the interaction) and the system consisting of some two-level atoms as well during propagation and amplification through a gaseous medium it leads to new qualitative phenomena.

3. Comparison of the probability of the first-order processes at various regimes of switching on the interaction

As it was shown in item 1 the calculation of the probability of the resonant radiation scattering on a two-level system is reduced to the calculation of ES emission. For this purpose in the expression (8) it is necessary to change the negative component of the dipole moment of the free atom by the negative frequency part of the ES dipole moment D_{ik} in the case of adiabatic switching on the interaction and by D'_{ik} in the case of sudden switching on the interaction, respectively. For absorption processes it is necessary to substitute the positive frequency parts of the corresponding dipole moments of ES D^{+}_{ik} and D'^{+}_{ik} in the expression (8) and make use of the formulas (11) and (12) for emission and absorption, respectively. As it is seen from the previous item the dipole moments D_{ik} and D'_{ik} have the components at the resonant and the side $(\omega \pm \Omega)$ frequencies. Fig. 1. represents the dependence of the side frequencies upon for $\Delta > 0$ and $\Delta < 0$. It is obvious that the radiations at different frequencies do not interfere with each other and

should be calculated separately. As to emission and adsorption at the same frequency, they can compensate each other in stimulating processes if $D_{ik}^{+} = D_{ik}^{-}$ or $D_{ik}'^{+} = D_{ik}'^{-}$. As a result of there remains only a spontaneous emission. If $D_{ik}^{+} \neq D_{ik}^{-}$ or $D_{ik}'^{+} \neq D_{ik}'^{-}$ there is no complete compensation of stimulated processes of emission and absorption and to obtain a resulting radiation one needs to subtract one from another. The probability of absorption is conditionally considered to be negative. These probabilities are set on the negative direction of the ordinate-axis in Fig. 2 and 3. The stated rules take place during the radiation processes of the first order with respect to a weak field. The radiative processes (termed as parametric ones considered in Ref. [11]) of the second order can also be considered by using the qusiclassical theory of emission. They may become essential in forward scattering . The effect of the processes of the second-order with respect to a weak field is not considered in the present paper.

We begin by considering the case of adiabatic switching on the interaction. The probability of spontaneous emission at the frequency $\omega' = \omega$ is coherent one and given by the following expression at $\alpha < 1$ for the transition $\Phi_1 \rightarrow \Phi_2$

$$dW_{11} = \frac{\alpha^2}{4}\left(1-\alpha^2\right) dW_{sp}(\omega) \tag{26}$$

The probabilities of emission and absorption at the side frequencies $\omega \pm \Omega$ are noncoherent ones and given by the following formulas for the transition $\Phi_1 \rightarrow \Phi_2$ at $\Delta > 0$

$$dW_{21}\left(\omega-\Omega\right) = \frac{\alpha^4}{16} dW_{sp}(\omega-\Omega)\left[n(\omega-\Omega)+1\right] \tag{27}$$

$$dW_{21}\left(\omega+\Omega\right) = -\left(1-\frac{\alpha^2}{2}\right) dW_{sp}(\omega+\Omega) n(\omega+\Omega). \tag{28}$$

Under the condition $\Delta < 0$ in the expressions (27,28) the coefficients and signs are changed over

$$dW_{21}\left(\omega-\Omega\right) = \left(1-\frac{\alpha^2}{2}\right) dW_{sp}(\omega-\Omega)\left[n(\omega-\Omega)+1\right] \tag{29}$$

$$dW_{21}\left(\omega+\Omega\right) = -\frac{\alpha^4}{16} dW_{sp}(\omega+\Omega) n(\omega+\Omega). \tag{30}$$

These processes are both noncoherent ones.

In the case of $\alpha^2 \gg 1$ the probability of the coherent emission of quanta with the

frequency is coherent and given by the expression

$$dW_{11} = \frac{1}{4}\left(1 - \frac{1}{\alpha^2}\right) dW_{sp}. \tag{31}$$

The probabilities of emission and absorption at the side frequencies for the transition $\Phi_1 \to \Phi_2$ are noncoherent and equal to

$$dW_{21}\left(\omega - \Omega\right) = \frac{1}{4}\left(1 - \frac{2sign\Delta}{\alpha}\right) dW_{sp}(\omega - \Omega)\left[n(\omega - \Omega) + 1\right] \tag{32}$$

$$dW_{21}\left(\omega + \Omega\right) = -\frac{1}{4}\left(1 + \frac{2sign\Delta}{\alpha}\right) dW_{sp}(\omega + \Omega) n(\omega + \Omega). \tag{33}$$

For comparison convenience the curves for $\omega \pm \Omega$ are shown in Fig. 1 in dependence upon ω at $\Delta > 0$ and $\Delta < 0$. Fig. 2 shows the probabilities of coherent and noncoherent processes for adiabatic switching on in terms dW_{sp} depending upon the frequency at $\alpha < 1$ on the left side of Fig. 2 and on the right one at $\alpha > 1$. The straight line above the abscissa-axis represents emission and that below the axis represents absorption.

Let us turn now to the probabilities of emission and absorption in the case of sudden switching on the interaction. In this case, as it follows from the relations (1) the separation of coherent and noncoherent processes is getting rather complicated. It is easy to see that coherent processes will involve the processes not only at a non-shifted frequency but also at the side frequencies of $\omega \pm \Omega$. Analogous phenomena will be also observed during noncoherent processes where besides of noncoherent processes for shifted frequency there appears noncoherent scattering at a non-shifted frequency.

First consider coherent processes, i.e. emission and absorption at $\Phi'_{1,2} \to \Phi'_{1,2}$ when the wave function of an ES does not change. By using the formulas (17,18) for the probabilities of the radiative first order processes with respect to a weak field we have at $\alpha^2 \ll 1$:

$$dW'_{11}(\omega) = \frac{\alpha^2}{4}\left(1 - \alpha^2\right) dW_{sp}(\omega) at \Delta > 0 and \Delta < 0 \tag{34}$$

$$dW'_{11}(\omega - \Omega) = \frac{\alpha^6}{64} dW_{sp}(\omega - \Omega) at \Delta > 0; \tag{35}$$

$$dW'_{11}(\omega - \Omega) = \frac{\alpha^2}{4}\left(1 - \frac{\alpha^2}{2}\right) dW_{sp}(\omega - \Omega) at \Delta > 0; \tag{36}$$

$$dW'_{11}(\omega + \Omega) = \frac{\alpha^2}{4}\left(1 - \frac{\alpha^2}{2}\right) dW_{sp}(\omega + \Omega) at \Delta > 0; \tag{37}$$

$$dW\prime_{11}(\omega+\Omega)=\frac{\alpha^6}{64}dW_{sp}(\omega+\Omega)at\Delta<0. \tag{38}$$

The corresponding probabilities for $\alpha^2 \gg 1$ with taking into account the formulas (22), (24) and (25) take the following form:

$$dW'_{11}(\omega)=\frac{1}{4\alpha^2}\left(1-\frac{1}{\alpha^2}\right)dW_{sp}(\omega), \tag{39}$$

$$dW'_{11}(\omega+\Omega)=\frac{1}{16}\left(1+\frac{2sign\Delta}{\alpha}\right)dW_{sp}(\omega+\Omega), \tag{40}$$

$$dW'_{11}(\omega-\Omega)=\frac{1}{16}\left(1-\frac{2sign\Delta}{\alpha}\right)dW_{sp}(\omega-\Omega). \tag{41}$$

Analogous expressions (34-41) take place also for the $\Phi'_2 \rightarrow \Phi'_2$ transitions which follows from the expressions (16) and (22). Determine the probabilities of noncoherent processes of the first order with respect to the weak field by proceeding from the expressions (17), (18), (23). The probability of the non-shifted scattering is independent of the sign of Δ and has the following form:

$$dW'_{21}(\omega)=\frac{\alpha^4}{4}\left(1-\alpha^2\right)dW_{sp}(\omega). \tag{42}$$

The probabilities of emission with the frequency $\omega-\Omega$ and absorption for $n>\frac{\alpha^4}{16}$ with the frequency $\omega+\Omega$ have the following form at $\Delta>0$:

$$dW'_{21}(\omega+\Omega)=\left[-n(\omega+\Omega)+\frac{\alpha^4}{16}\right]dW_{sp}(\omega+\Omega) \tag{43}$$

$$dW'_{21}(\omega-\Omega)=\frac{\alpha^4}{16}\left[n(\omega-\Omega)+1\right]dW_{sp}(\omega-\Omega). \tag{44}$$

The probabilities of emission of a quantum with the frequency $\omega+\Omega$ and absorption for $n>\frac{\alpha^4}{16}$ of a photon with the frequency $\omega-\Omega$ have the following form at $\Delta<0$:

$$dW'_{21}(\omega+\Omega)=\frac{\alpha^4}{16}\left[n(\omega)+1\right]dW_{sp}(\omega+\Omega) \tag{45}$$

$$dW'_{21}(\omega-\Omega)=\left[-n(\omega-\Omega)+\frac{\alpha^4}{16}\right]dW_{sp}(\omega-\Omega). \tag{46}$$

The probabilities of the noncoherent processes of the first-order with respect to the weak field at the $\Phi'_1 \rightarrow \Phi'_2$ transition with taking into account the formulas (23) - (25) for $\alpha^2 \gg 1$ have the following form:

$$dW'_{21}(\omega)=\frac{1}{4}\left(1-\frac{1}{\alpha^2}\right)dW_{sp}(\omega)at\Delta>0and\Delta<0 \tag{47}$$

$$dW'_{21}(\omega+\Omega) = \left[-\left(\frac{sign\Delta}{4\alpha}+\frac{1}{2\alpha^2}\right)n(\omega+\Omega)+\frac{1}{16}\left(1-\frac{2}{\alpha^2}\right)\right]dW_{sp}(\omega+\Omega) \tag{48}$$

$$dW'_{21}(\omega-\Omega) = \left[\left(\frac{sign\Delta}{4\alpha}-\frac{1}{2\alpha^2}\right)n(\omega-\Omega)+\frac{1}{16}\left(1-\frac{2}{\alpha^2}\right)\right]dW_{sp}(\omega-\Omega). \tag{49}$$

So long as there exist the relation $D'_{12} = D'^{*}_{12}$ it is easy to obtain corresponding expressions for the first order radiation processes when at t=0 wave function of ES is equal to ψ_2.

The dependencies of the probabilities of coherent and noncoherent processes during sudden switching are presented in Fig. 3; for $\alpha < 1$ on the left and for $\alpha > 0$ on the right.

It is easy to see that the maximum of the processes of the first order with respect to a weak field occurs at the value of $\alpha \approx 1$. Spontaneous processes of emission are termed as resonance fluorescence and in linear regime were considered in 30s [3-6]. In nonlinear regime they became to be under consideration in 50s due to the development of laser technique. In one of the last experimental work provided by H. Walther's laboratory (see [12] and the references on the previous publications therein) they succeeded in measuring the bandwidth of fluorescence line to phenomenal accuracy (about few Hz) by using the heterodyne method for frequency measuring [12]. The processes of a probe field absorption were begun to be studied due to laser creation in 50s (see, related reference in [8]). The process of the side line amplification in the optical range at the $\omega-\Omega$ frequency, called by authors three-photon radiation,and $\alpha \sim 1$ in the experiments on propagation of resonance radiation through a gaseous medium was discovered for the first time and investigated by Movsesyan M. et al in 1968 [13]. The detailed investigation in this field on the atoms of Na and comparison with a theory of B.R. Mollow [14] was performed in [15].

4. Conclusion

As it has already been noted the formulas given in the paper can be of great use in various fields of atomic physics and quantum theory of information recording, storage and transfer. It is desirable to carry out more detailed research using quantum theory of emission that in some cases can result in new phenomena. The expressions given in this paper for probabilities of first-order emission with respect to a weak field, photons

of that are different from those of strong resonant field, allow to calculate the width of ES quasienergy levels.

If we define the level width of unperturbed atom by $\Gamma = \int dW_{sp}$, then the width of ES which is in Φ_1 state will be determined by the probabilities (26)-(32). If $n(k', e') = 0$, then in the case of adiabatic switching the width of wave function Φ_1 will be equal to

$$\Gamma(ES) = \frac{\sqrt{1+\alpha^2} - sign\Delta}{2\sqrt{1+\alpha^2}}\Gamma = n_2\Gamma, \tag{50}$$

where n_2 is a weight of ψ_2 state in Φ_1 function, i.e. it determines the probability of electron being in the upper level of unperturbed atom. The decay of ES occurs owing to spontaneous decay of unperturbed atom upper level. At $\alpha_2 \gg 1$ the Φ_1 state will be decaying with the width equal to $\Gamma(ES) = \Gamma/2$ as the electron spends half of time in the upper level, i.e. in ψ_2.

At $\alpha \to 0$ $\Phi_1 \to \psi_1$ (for adiabatic switching at $\Delta > 0$) and naturally in this case $\Gamma(ES) = 0$. However, at the presence of weak photons different from photons of resonant pumping either in direction or in polarization there will also arise stimulated widths which as spontaneous ones are dependent on the state in which ES is.

The decoherence problem of ES has attracted great attention of specialists working in the field of quantum memory and information transfer. These problems can be investigated for our case with the decay probabilities given in this paper.

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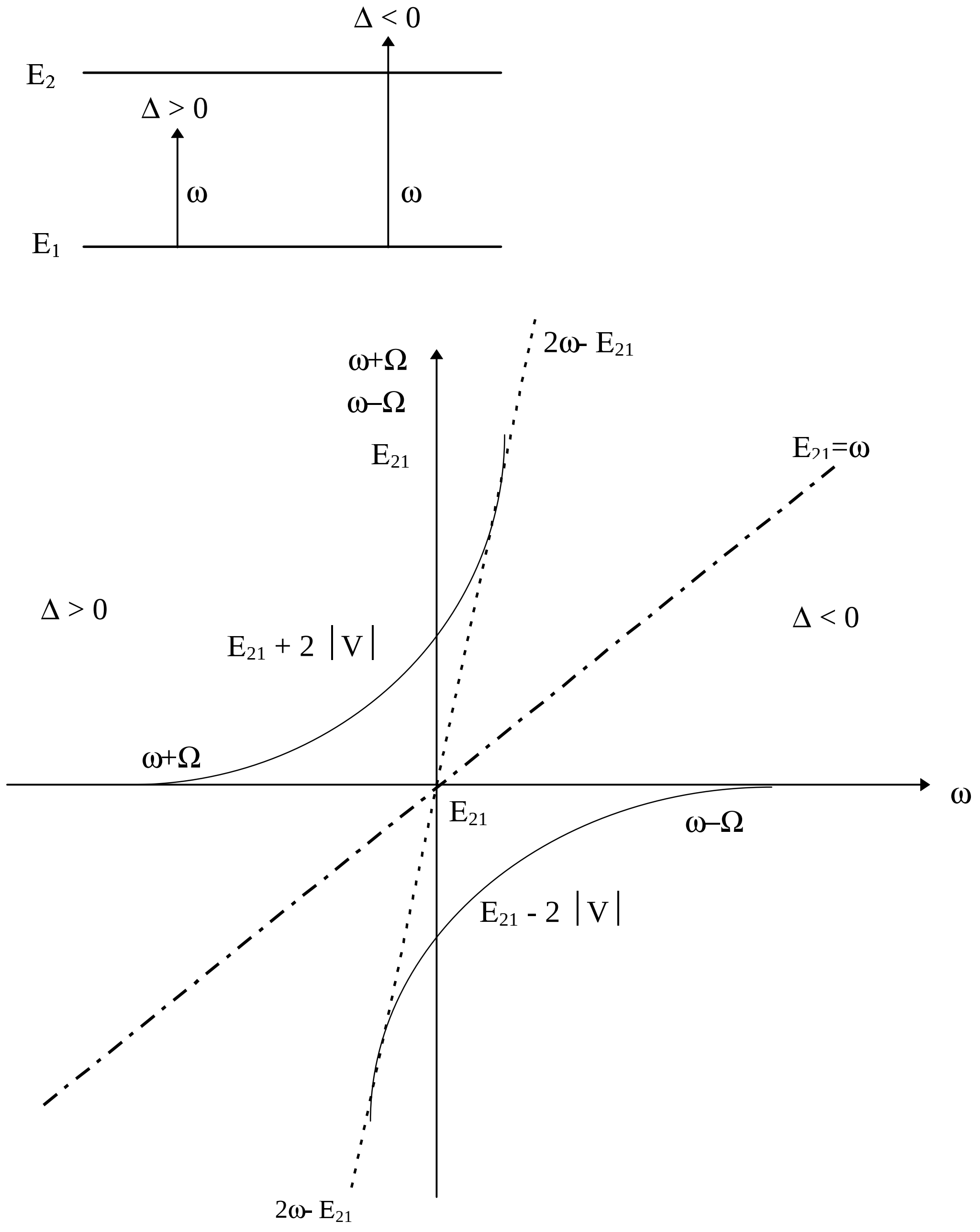
Figure captions

Fig. 1. The dependence of $\omega+\Omega$ (upper curve) and $\omega-\Omega$ (lower curve) frequencies on is presented in Fig. 1. The curves $\omega\pm\Omega$ are symmetric with respect to $E_{21}=\omega$ line (dash-dot). At $\alpha^2\to 0$ the $\omega-\Omega$ line at $\Delta<0$ is approaching to abscissa axis and at $\Delta>0$ to $2\omega-E_{21}$ (dot). At $\alpha^2\ll 1, \omega-\Omega=\omega-|\Delta|\,(1+\alpha^2)$, $\omega+\Omega=\omega+|\Delta|\,(1+\alpha^2)$. At $\alpha^2\gg 1$, $\omega-\Omega=\omega-|2V|-\frac{\Delta}{2\alpha}$; $\omega+\Omega=\omega+|2V|+\frac{\Delta}{2\alpha}$. The scheme of two-level atom interacting with resonant field at $\Delta>0$ and $\Delta<0$ is illustrated in the upper part on the left.

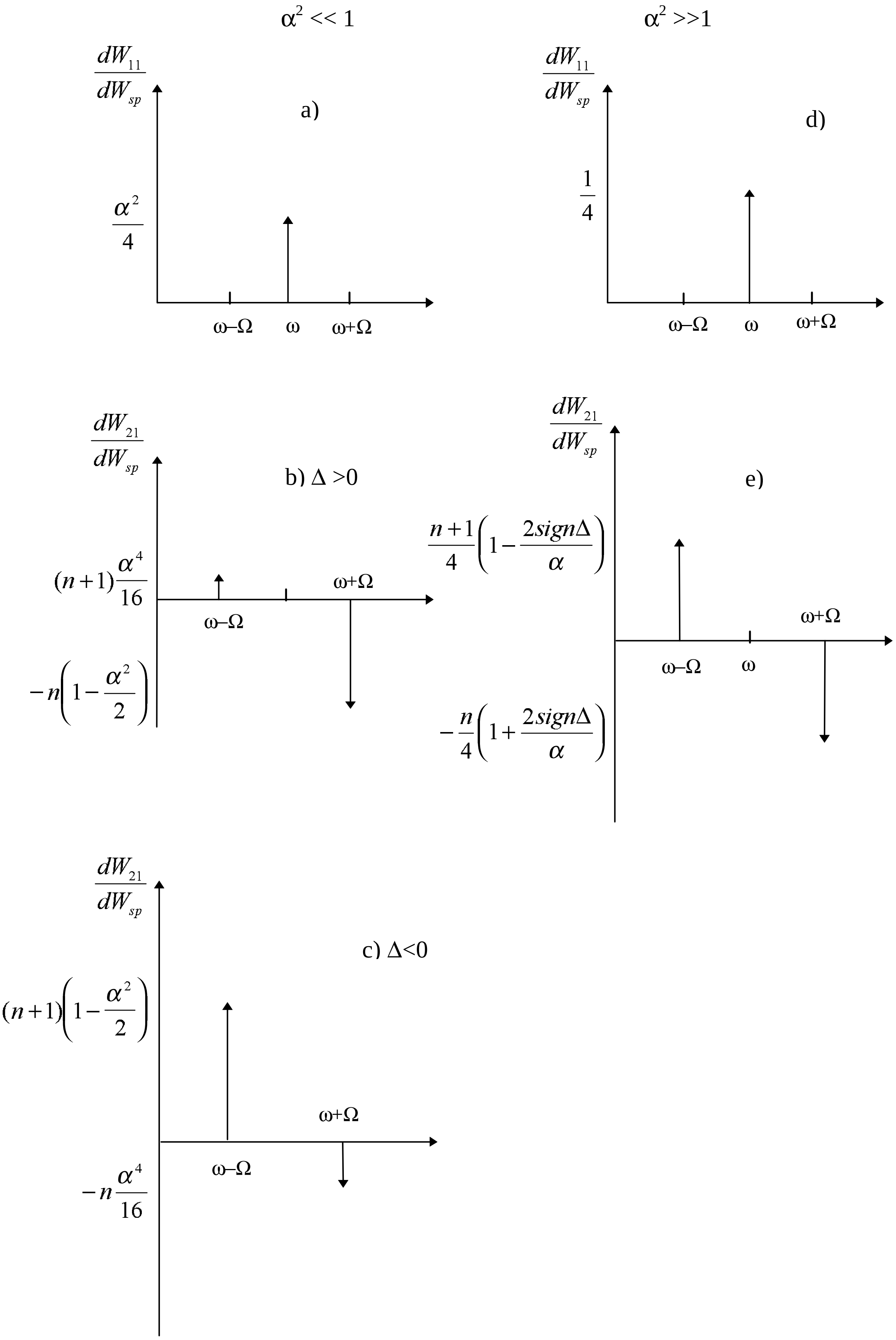
Fig. 2. The graphs for radiative processes probabilities at adiabatic switching on the interaction are presented. Frequencies are set on abscissa-axis. The probabilities of emission (above abscissa-axis) and absorption (below abscissa-axis) are set on ordinate-axis. The probabilities are given in units of spontaneous probabilities at corresponding frequencies. The left column of graphs corresponds to the parameter of $\alpha^2\ll 1$; the right one to $\alpha^2\gg 1$. The upper graphs a) and d) correspond to coherent processes at which the wave function of ES does not change in the processes of emission and absorption of weak probe field. The graphs b), c) and e) correspond to the probabilities of noncoherent processes at $\Phi_1\to\Phi_2$ transition. To obtain corresponding probabilities for resonant fluorescence (spontaneous processes) the probabilities for spontaneous parts from the graphs b), c) and e) should be added to the graphs a) and d) at $\alpha^2\gg 0$ and $\alpha^2\ll 0$ correspondingly. The graphs have an illustrative character. Fig. 3. The graphs for radiative processes probabilities for sudden switching on the interaction are presented. Frequencies are set on abscissa-axis. The probabilities of emission (above abscissa-axis) and absorption (below abscissa-axis) are set on ordinate-axis. The probabilities are given in units of spontaneous probabilities at corresponding frequencies. The left column of graphs corresponds to the parameter of $\alpha<1$, the right one to $\alpha>1$. The upper graphs a), b) and d) correspond to coherent processes at which the wave function of ES does not change in the processes of emission and absorption of weak probe field. The graphs c), e) and f) correspond to the $\Phi_1'\to\Phi_2'$ processes. It is accepted in the graph c) that $n>\frac{\alpha^4}{16}$ and $\Delta>0$. In the case of $\Delta<0$ and $n>\frac{\alpha^4}{16}$ the directions of vertical lines and their values should be changed over, i.e. the quantum with $\omega-\Omega$ frequency will

be absorbed in accordance with formula (31′); the quantum with $\omega + \Omega$ frequency will be emitted according to formula (30′). These lines are dotted ones in graphic c). It is accepted on graph e) and f) that $\frac{1}{16} > \frac{n}{4\alpha}$. The relative height of vertical lines on graph e) corresponds to the conditions $\Delta > 0$ and on graphic f) to $\Delta < 0$. If the opposite inequality is satisfied i.e. $\frac{1}{16} < \frac{n}{4\alpha}$, then the vertical line at frequency $\omega - \Omega$ at $\Delta < 0$ on graph f) will be below abscissa-axis and the vertical line on graph e) for $\omega + \Omega$ frequency for $\Delta > 0$ will be below abscissa-axis. To obtain the corresponding probabilities for resonant fluorescence (spontaneous processes) the probabilities for spontaneous parts of noncoherent processes should be added to the probabilities on graphs a) and d). The graphs have illustrative character. It is followed from the figure that at $\alpha^2 \gg 1$ that Rayleigh scattering is completely noncoherent. The probabilities of spontaneous emissions at side frequencies consist of coherent and noncoherent components equal to each other (compare with figures in [5], where figure b is not correct). Fig. 4 represents spontaneous induced emission processes of entangled system (resonance fluorescence). The graphs a), b), and e) correspond to sudden adiabatic resonance. The graphs c), d) and f) correspond to adiabatic one. Fig. 5 shows absorption and emission of a probe field by entangled system for sudden and adiabatic switching. The graphs a), b), e) and f) correspond to sudden switching. The graphs c), d) and g) correspond to adiabatic one.



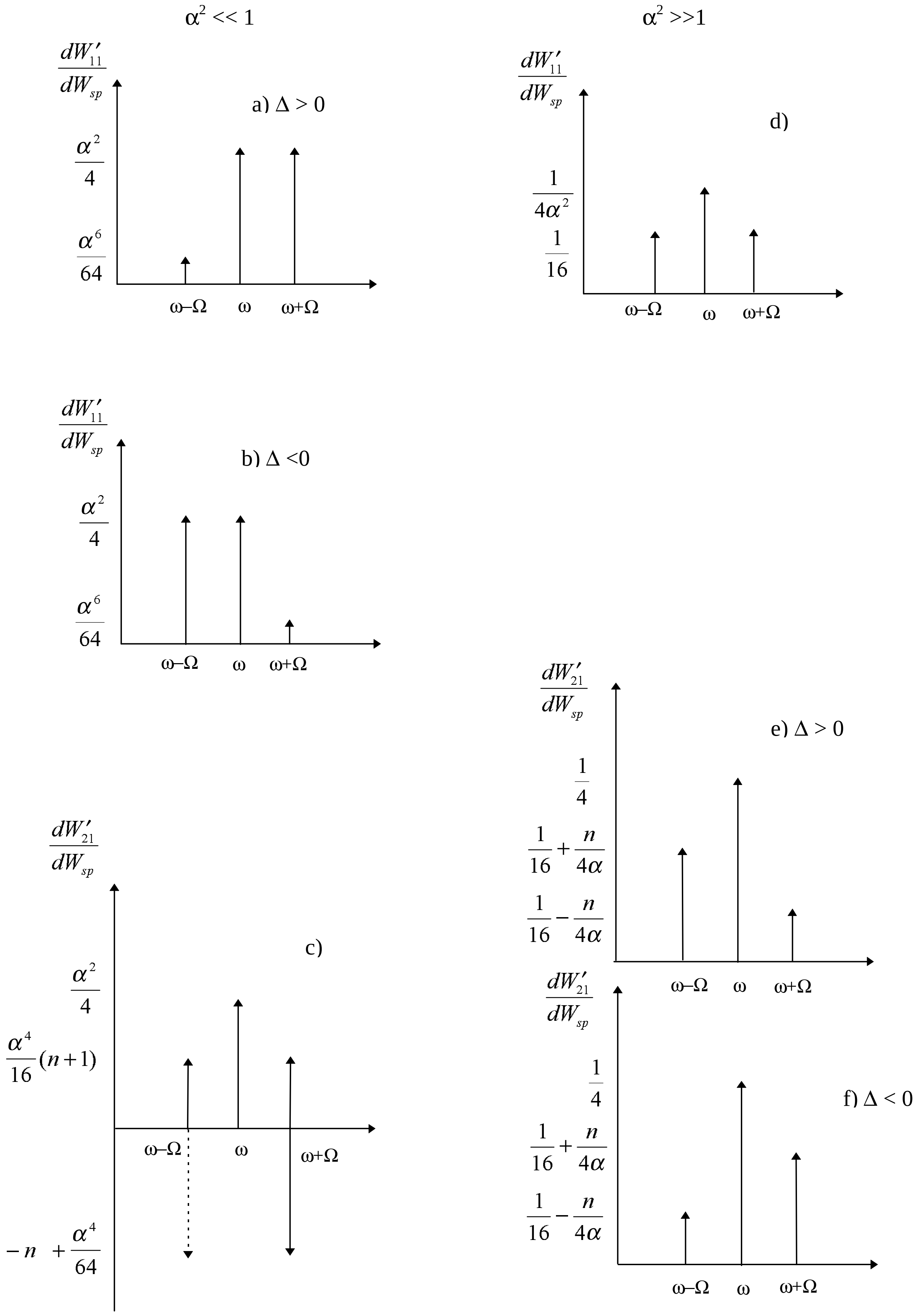
Δ < 0
E2
Δ > 0
ω
ω
E1
2ω- E21
ω+Ω
ω–Ω
E21
E21=ω
Δ > 0
Δ < 0
E21 + 2 │V│
ω+Ω
ω
E21
ω–Ω
E21 - 2 │V│
2ω- E21

Fig. 1



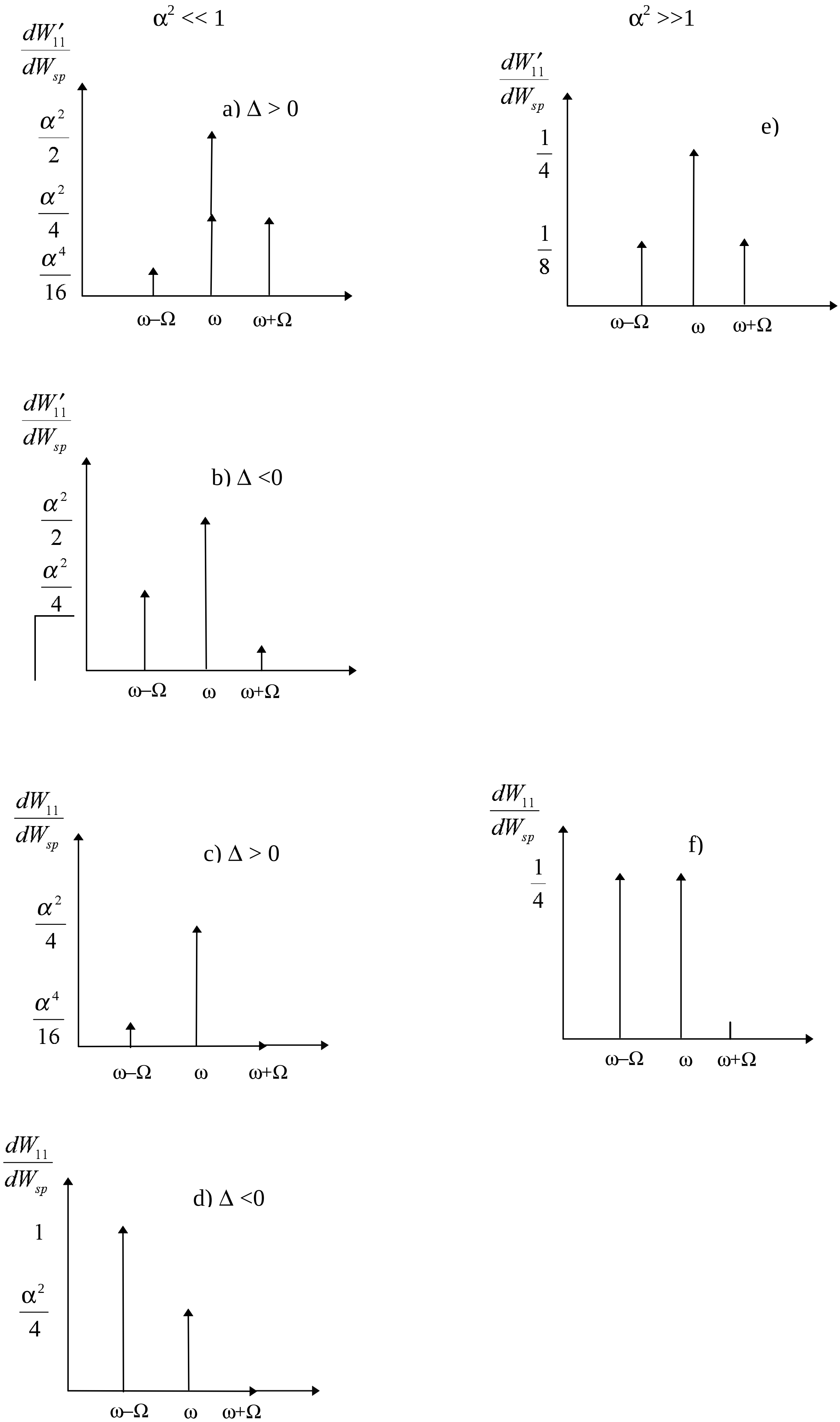
α² << 1
α² >>1
$\frac{dW_{11}}{dW_{sp}}$
a)
$\frac{\alpha^2}{4}$
ω–Ω
ω
ω+Ω
$\frac{dW_{11}}{dW_{sp}}$
d)
$\frac{1}{4}$
ω–Ω
ω
ω+Ω
$\frac{dW_{21}}{dW_{sp}}$
b) Δ >0
$(n+1)\frac{\alpha^4}{16}$
ω+Ω
ω–Ω
$-n\left(1-\frac{\alpha^2}{2}\right)$
$\frac{dW_{21}}{dW_{sp}}$
e)
$\frac{n+1}{4}\left(1-\frac{2sign\Delta}{\alpha}\right)$
ω+Ω
ω–Ω
ω
$-\frac{n}{4}\left(1+\frac{2sign\Delta}{\alpha}\right)$
$\frac{dW_{21}}{dW_{sp}}$
c) Δ<0
$(n+1)\left(1-\frac{\alpha^2}{2}\right)$
ω+Ω
ω–Ω
$-n\frac{\alpha^4}{16}$

Fig. 2



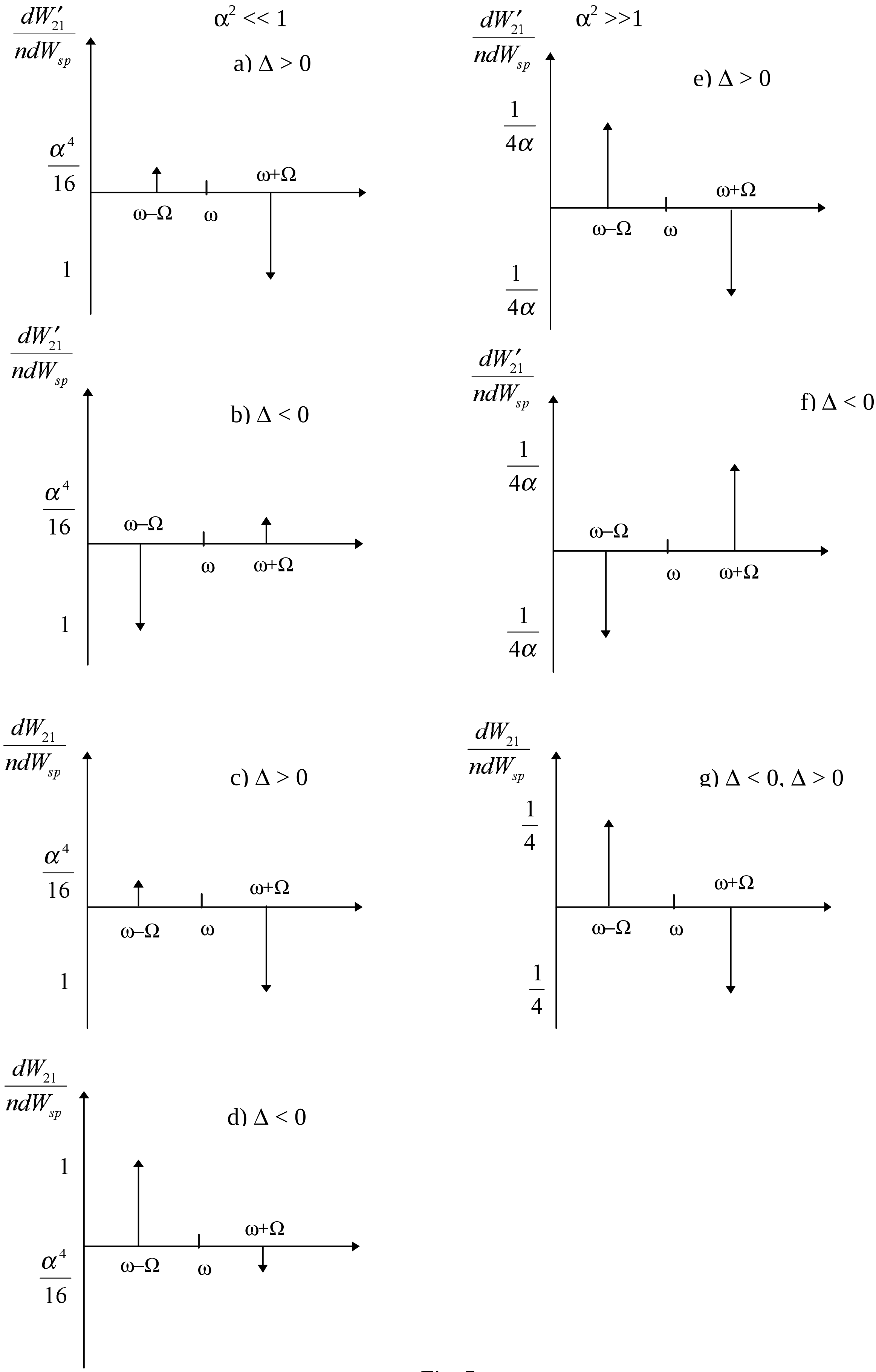
$\alpha^2 << 1$
$\alpha^2 >> 1$
$\frac{dW'_{11}}{dW_{sp}}$
a) $\Delta > 0$
$\frac{\alpha^2}{4}$
$\frac{\alpha^6}{64}$
ω–Ω
ω
ω+Ω
$\frac{dW'_{11}}{dW_{sp}}$
d)
$\frac{1}{4\alpha^2}$
$\frac{1}{16}$
ω–Ω
ω
ω+Ω
$\frac{dW'_{11}}{dW_{sp}}$
b) $\Delta < 0$
$\frac{\alpha^2}{4}$
$\frac{\alpha^6}{64}$
ω–Ω
ω
ω+Ω
$\frac{dW'_{21}}{dW_{sp}}$
e) $\Delta > 0$
$\frac{1}{4}$
$\frac{1}{16}+\frac{n}{4\alpha}$
$\frac{1}{16}-\frac{n}{4\alpha}$
ω–Ω
ω
ω+Ω
$\frac{dW'_{21}}{dW_{sp}}$
c)
$\frac{\alpha^2}{4}$
$\frac{\alpha^4}{16}(n+1)$
ω–Ω
ω
ω+Ω
$-n \quad + \frac{\alpha^4}{64}$
$\frac{dW'_{21}}{dW_{sp}}$
f) $\Delta < 0$
$\frac{1}{4}$
$\frac{1}{16}+\frac{n}{4\alpha}$
$\frac{1}{16}-\frac{n}{4\alpha}$
ω–Ω
ω
ω+Ω

Fig. 3



α² << 1
α² >>1
dW′₁₁/dW_sp
a) Δ > 0
α²/2
α²/4
α⁴/16
ω–Ω
ω
ω+Ω
e)
1/4
1/8
b) Δ <0
dW₁₁/dW_sp
c) Δ > 0
f)
d) Δ <0
1

Fig. 4



dW'21 / ndWsp
α2 << 1
a) Δ > 0
α4 / 16
1
ω–Ω
ω
ω+Ω
b) Δ < 0
c) Δ > 0
dW21 / ndWsp
d) Δ < 0
α2 >>1
e) Δ > 0
1 / 4α
f) Δ < 0
g) Δ < 0, Δ > 0
1 / 4

Fig. 5